SURFACES OF DISCONTINUITY IN A MAGNETIZED MEDIUM

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Features of surfaces of discontinuity in a magnetized medium are considered. It is shown that the polarizing force at interfaces may be directed towards the medium with higher permittivity. It is proved that there exists in an absolutely conducting magnetized medium plane-polarizing discontinuities at which the density of the medium remains invariant.

In this article, features of the boundary conditions on surfaces of discontinuity in a medium that may be isotropic and nonhomogeneously magnetized in an induced electromagnetic field are considered.

We obtain for the case of an ideally conducting medium ($E \sim c^{-1}vB$), whose motion is determined by eight variables (ρ, p, **v**, H), the eight boundary conditions [1]

$$
\begin{gathered}
[\rho v_n]=0,\ [\rho v_n \mathbf{v}_\tau-(4\pi)^{-1}\mu H_n \mathbf{H}_\tau]=0 \\
[\rho v_n^2+p-\rho^2 u_\rho-(4\pi)^{-1}\mu H_n^2]=0 \\
[\rho v_n(^1/_2 v^2+w-u-\rho u_\rho+Tu_T)+(4\pi)^{-1}(v_n \mu H^2-\mu H_n(\mathbf{H}\cdot\mathbf{v}))]=0 \\
[\mu H_n]=0,\ \mu H_n[\mathbf{v}_\tau]=[v_n \mu \mathbf{H}_\tau] \\
u=(4\pi\rho)^{-1}\int_0^H \mu(\rho, T, H)\, H dH, \qquad u_\rho \equiv \frac{\partial u}{\partial \rho}, \qquad u_T \equiv \frac{\partial u}{\partial T}
\end{gathered} \tag{1}
$$

Here $\mathbf{v}_\tau$ and $\mathbf{H}_\tau$ are the velocity and field components tangent to the surface of discontinuity, w is the enthalpy of the medium in the absence of an electromagnetic field, $[\mathbf{a}] \equiv \mathbf{a}_2-\mathbf{a}_1$, where $\mathbf{a}_1$ and $\mathbf{a}_2$ are the values of **a** before and after the discontinuity, respectively.

The last condition in Eqs. (1) was obtained from the continuity of the tangential component $\mathbf{E}_\tau$ and the discontinuity $\mathbf{H}_\tau$ determines the surface current $\mathbf{i}=c(4\pi)^{-1}\times([\mathbf{H}_\tau]\times\mathbf{n})$.

For a nonconducting medium ($[\mathbf{H}_\tau]=0$) we also obtain eight* conditions:

$$
\begin{gathered}
[\rho v_n]=0, \quad [\rho v_n \mathbf{v}_\tau]=0 \\
[\rho v_n^2+p-\rho^2 u_\rho-(4\pi)^{-1}\mu H_n^2]=0 \\
[\rho v_n(^1/_2 v^2+w-u-\rho u_\rho+Tu_T)]=0 \\
[\mu H_n]=0, \quad [\mathbf{H}_\tau]=0
\end{gathered} \tag{2}
$$

When μ = const, the conditions of Eqs. (2) reduce to ordinary gasdynamical conditions and conditions on a magnetic field that does not interact with the medium, while the conditions of Eqs. (1) reduce to the conditions on the discontinuities that are considered in magnetic hydrodynamics.

We will consider, following the classification accepted in magnetic hydrodynamics, the features that facilitate magnetization of the medium and introduce these features into the characteristics of each type of discontinuity.

Contact discontinuities are surfaces of discontinuity consisting of flow lines through which no matter flows ($v_n=0$), though there exists a magnetic flux ($H_n \neq 0$). We obtain for them from Eqs. (1),

*As in Russian original; only six conditions are provided – Publisher.

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$$[\mathbf{H}_\tau] = 0, \quad [v_\tau] = 0, \quad [\mu H_n] = 0, \quad [p] = [\rho^2 u_\rho + (4\pi)^{-1}\mu H_n^2] \tag{3}$$

Here the discontinuity $[\rho]$ remains arbitrary.

All the conditions of Eqs. (3), other than $[v_T] = 0$, must be satisfied, as follows from Eq. (2), at contact discontinuities in a conducting magnetized medium.

It follows from Eqs. (3) that pressure is not a continuous variable at contact discontinuities of a magnetized medium (they may sometimes be the interfaces of two immiscible media), and the pressure discontinuity is determined by the difference in permittivities from one side to the other.

If μ is constant in the media on both sides of the discontinuity, we obtain from Eqs. (3)

$$[p] = -\frac{[\mu]}{8\pi}\left(\frac{B_n^2}{\mu_1\mu_2} + H_\tau^2\right) \tag{4}$$

It follows from Eq. (4) that pressure is always greater in the medium with lesser permittivity, so that a normal force towards the medium with lesser permittivity acts at the contact discontinuity.

However, the picture may change if we take into account magnetostriction forces.

For example, for media magnetized according to the Clausius–Mossoti law$[(\mu - 1)T/\rho\mu = \text{const}]$, we find from Eqs. (3),

$$[p] = \frac{H_\tau^2}{8\pi}[\mu(\mu - 2)] = \frac{H_\tau^2}{8\pi}(\mu_2 - \mu_1)(\mu_2 + \mu_1 - 2)$$

It therefore follows that in paramagnetic media ($\mu > 1$), the pressure is greater towards the discontinuity where the magnetic permeability is greater, so that the direction of the polarizing force acting on the contact discontinuity varies in the opposite direction.

The pressure discontinuity is determined solely by the value of the tangential field component.

For a magnetically saturated medium, when we may let $\mu = 1 + 4\pi H^{-1}M(\rho, T)$, where M is the magnetization function, we obtain from Eqs. (3)

$$[p] = \frac{B_n^2}{8\pi}\left[\left(1 - \frac{1}{\mu}\right)^2\right] + \left[\rho^2 H \frac{\partial}{\partial\rho}\left(\frac{M}{\rho}\right)\right]$$

Hence, if M linearly depends on ρ, which is usually assumed on the basis of an elementary kinetic analysis for gases, pressure in the paramagnetic medium will be greater where μ is less, but the pressure discontinuity in this case will be determined solely by the normal field component.

But if we do not take into account magnetostriction effects $[M = M(T)]$, which sometimes act for dropping media [2], we will have

$$[p] = -\frac{[\mu]}{4\pi}\left(H_\tau^2 + \frac{B_n^2(\mu_1 + \mu_2)}{2\mu_1^2\mu_2^2}\right)$$

Evidently, here the polarizing force acts towards the medium with lesser permittivity and is determined by both the normal and tangential magnetic field components.

The polarizing forces we have been examining play a substantial role in the stability of interfaces of magnetized media.

Since the density discontinuity at interfaces is arbitrary, the two last conditions of Eqs. (3) allow us to completely determine p_2 and T_2 in terms of variables prior to the discontinuity if we know the function $\mu = \mu(\rho, T, H)$.

The tangential discontinuities ($v_n = 0$, $H_n = 0$) are interfaces in the tangential magnetic field. We obtain from Eqs. (3) for these discontinuities $[p] = [\rho^2 u\rho]$, where the discontinuities $[v_T]$, $[H_T]$, and $[\rho]$ may have arbitrary values (for a nonconducting medium only $[v_T]$ and $[\rho]$ are arbitrary, whereas $[H_T] = 0$).

Tangential discontinuities thus can carry a surface current (in a conducting medium) into a vortex sheet.

The pressure discontinuity on the tangential discontinuities is therefore determined not only by the presence of magnetic permeabilities, but also by the difference in the tangential field components.

For example, in weak fields [$\mu = \mu(\rho, T)$] we have for the tangential discontinuities

$$[p] = \frac{1}{8\pi}\left[H_\tau^2\left(\rho\frac{\partial\mu}{\partial\rho} - \mu\right)\right]$$

It therefore follows that if there is always a pressure discontinuity in a nonmagnetic medium in the presence of a surface current, magnetostriction forces in a magnetized medium are able to entirely eliminate this discontinuity (for example, when $\mu/\rho = \text{const}$).

Let us now consider such discontinuities as the Alfven type and shock waves through which matter flows, i.e., such that $v_n \neq 0$. Here, eliminating the discontinuity $[v_\tau]$ by means of the second relation of Eqs. (1), we write the system (1) by letting $m_n \equiv \rho v_n \neq 0$, in the form

$$[\rho v_n] = 0, \quad [\mu H_n] = 0, \quad [m_n^2\rho^{-1} + p - \rho^2 u_\rho - (4\pi)^{-1}\mu H_n^2] = 0 \tag{5}$$
$$\left[\frac{m_n^2}{2\rho^2} + w - \rho u_\rho - u + Tu_T + \frac{\mu H_\tau^2}{4\pi\rho} - \frac{\mu^2 H_n^2 H_\tau^2}{32\pi^2 m_n^2}\right] = 0$$
$$\left[\frac{\mu \mathbf{H}_\tau}{v_n}\left(v_n^2 - \frac{\mu H_n^2}{4\pi\rho}\right)\right] = 0$$

and the system (2) for a nonconducting medium, in the form

$$[\rho v_n] = 0, \quad [\mu H_n] = 0, \quad [m_n^2\rho + p - \rho^2 u_\rho - (4\pi)^{-1}\mu H_n^2] = 0 \tag{6}$$
$$\left[\frac{m_n^2}{2\rho^2} + w - \rho u_\rho - u + Tu_T\right] = 0$$
$$[\mathbf{v}_\tau] = 0, \quad [\mathbf{H}_\tau] = 0$$

Alfven discontinuities are surfaces of discontinuity, and the medium in passing through them will not experience a change in its density, i.e., we have $v_n \neq 0$, but $[\rho] = 0$.

We have for Alfven discontinuities in a magnetized conducting medium, using Eqs. (5),

$$[v_n] = 0, \quad [\mu H_n] = 0, \quad [p - \rho_1^2 u_\rho - (4\pi)^{-1}\mu H_n^2] = 0 \tag{7}$$
$$\left[w - \rho_1 u_\rho - u + Tu_T + \frac{\mu H_\tau^2}{4\pi\rho_1} - \frac{\mu^2 H_n^2 H_\tau^2}{32\pi^2 m_n^2}\right] = 0$$
$$\left[\mu \mathbf{H}_\tau\left(v_n^2 - \frac{\mu H_n^2}{4\pi\rho_1}\right)\right] = 0$$

It is well known [3] that an Alfven discontinuity in a nonmagnetic medium is a rotating discontinuity ($[H_\tau] = 0$), i.e., it possesses the property of circular polarization, and its velocity is equal to the Alfven velocity,

$$v_n = A_n \equiv \sqrt{\mu H_n^2 / 4\pi\rho_1}$$

This discontinuity may be plane-polarized in a medium with magnetization.

If the wave velocity is A_n, we have from the expression for A_n and from the first two relations of Eqs. (7) that $[\mu] = 0$ and $[H_n] = 0$. Here the variation in the magnitude of the vector $\mathbf{H}_\tau$ is, in general, related to a variation in the temperature of the medium by the relation $\mu_1 = \mu_2$, i.e.,

$$\mu\left(\rho_1, T_1, \sqrt{H_{n1}^2 + H_{\tau 1}^2}\right) = \mu\left(\rho_1, T_2, \sqrt{H_{n1}^2 + H_{\tau 2}}\right) \tag{8}$$

Consequently, when $v_n = A_n$, we have for given H_n three equations to determine T_2, p_2, and $H_{\tau 2}$ from Eqs. (7):

$$[\mu] = 0, \quad [p] = \rho_1^2 [u_\rho] \tag{9}$$
$$\left[w - \rho_1 u_\rho - u + Tu_T + \frac{\mu H_\tau^2}{8\pi\rho_1}\right] = 0$$

We note that the condition of Eq. (8) imposes a sufficiently strong constraint on the temperature variation and that if it does not hold when $T_1 \neq T_2$ and $H_{\tau 1} \neq H_{\tau 2}$ for nondecreasing total entropy in the discontinuity, only the ordinary rotary discontinuity may exist in which H_τ varies only in direction, while the variation of the remaining parameters always satisfies Eqs. (9). For example, when $\mu = \mu(\rho, T)$ Eq. (8) implies that $[T] = 0$. Then we have from Eqs. (9) that $[H_\tau] = 0$, $[p] = 0$, and $[w] = 0$, so that when $v_n = A_n$ in weak fields only an ordinary rotary discontinuity may occur.

If the velocity at which the medium passes through the discontinuity is not equal to the Alfven velocity, i.e., $v_n \neq A_n$, it will follow from the last relation of Eqs. (7) that the direction of $\mathbf{H}_\tau$ will not vary in the discontinuity, so that this wave is plane-polarized.

Letting $v_n/A_n \equiv \gamma$ we obtain from Eqs. (7) for given γ and B_n three equations for determining T_2, p_2, and $H_{\tau 2}$:

$$[(\mu - \mu_1\gamma^{-2}) H_\tau] = 0, \quad [p] = \rho_1^2 [u_\rho] + \frac{B_n^2}{4\pi}\left[\frac{1}{\mu}\right]$$
$$[w] = [\rho_1 u_\rho + u - Tu_T - (2\mu\gamma^2 - \mu_1) H_\tau^2 \gamma^{-2} (8\pi\rho_1)^{-1}]$$

These equations, consistent with the inequality $[s + u_T] \geq 0$, which expresses the fact that the total entropy of the medium in the jump is nondecreasing, are used to calculate the Alfven plane-polarized discontinuities existing only in a magnetized medium. An example of such a calculation in a saturated ideal gas ($\mu = 1 + 4\pi\rho K(\theta - T)H^{-1}$) is presented in [4].

It can be shown that when ρ = const, no simple wave propagating with finite velocity in a nonconducting magnetized medium exists.

Therefore, the systems of conditions following from Eqs. (6) when $[\rho] = 0$ imply that all the discontinuities are zero, so that Alfven discontinuities cannot exist in a nonconducting magnetized medium.

Shock waves are surfaces of discontinuity at which the conditions of Eqs. (5) and (6) hold in most general form, where $m_n \neq 0$ and $[\rho] \neq 0$.

Shock waves in a magnetized medium are plane-polarized.

The last three conditions of Eqs. (5) are used to determine T_2, p_2, and $H_{\tau 2}$, and the values of T_2 and p_2 are determined from Eqs. (6) in a nonconducting medium.

Magnetic induction is proportional to the density ρ ($\mathbf{B}/\rho$ = const) in a longitudinal shock wave ($B_n = 0$) in a conducting magnetized medium.

In a conducting magnetized medium there exist separate shock waves, as in a nonmagnetic medium.

Eliminating flow m_n from the fourth condition of Eqs. (5) by means of the third and last conditions, we obtain the shock adiabatic equation for a magnetized conducting medium in the form

$$[w] = [u + \rho u_\rho - Tu_T] + \left(\overline{\frac{1}{\rho}}\right)\left[p - \rho^2 u_\rho - \frac{\mu H_n^2}{4\pi}\right] - \frac{1}{4\pi}\left(\overline{\frac{\mu H_\tau}{\rho}}\right)[H_\tau] \tag{10}$$

where the bar denotes the mean value in the discontinuity ($\bar{a} \equiv \frac{1}{2}(a_2 + a_1)$).

The adiabatic equation (10) can be written in more compact form in terms of the total (hydrodynamic + electromagnetic) enthalpy and pressure, that is,

$$[w'] = \left(\overline{\frac{1}{\rho}}\right)\left[p' - \frac{\mu H_n^2}{4\pi}\right] - \frac{1}{4\pi}\left(\overline{\frac{\mu H_\tau}{\rho}}\right)[H_\tau] \tag{11}$$
$$p' \equiv p + p^e = p - \rho^2 u_\rho$$
$$w' \equiv w + U^e + \frac{p^e}{\rho} - \frac{HB}{4\pi\rho} = w - \rho u \rho - u + Tu_T$$

When $[H_T] = 0$, we obtain from Eqs. (10) and (11) a shock adiabatic equation for a nonconducting magnetized medium in a magnetic field.

When μ = const, Eqs. (10) and (11) become the well-known shock adiabatic equation for a nonmagnetic medium.

LITERATURE CITED

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